Indian J. Phys. 64B (5), 346-356 (1990)

Anticollapsars in extended manifolds : the second version of origin of white holes

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Received 5 October 1989, accepted 12 June 1990

Abstract : Anticollapsing objects are considered in the extended Kerr-Newman space-time manifold. The dependence of the blue shift on event horizons is found out, and expressions for maximal blue shift are given through global parameters of white and grey holes. A model of grey hole is proposed for explanation of cosmic gamma-bursts and gravitational waves bursts.

Keywords : White holes, grey holes, Kerr-Newman space-time, blue shift.

PACS Nos : 04.20. Cv, 04.30. + x

1. Introduction

The Kerr-Newman exact solution and its particular cases, which describe not only the black holes (Chandrasekhar 1983) but also their antipodes, white holes (Penrose 1979) equally, are some of the applications of general relativity (*GR*) to astrophysics. The black and white holes belong to the more wide class of theoretical relativistic objects with event horizons called otons by Zel'dovich and Novikov (1971).

It is known that together with collapsing regions in space-time manifolds (STM) which are described by exact solutions of *GR* anticollapsing ones must exist also both in the simplest Schwarzschild STM and in more rich STM having non-trivial structure (Hawking and Ellis 1973). An attempt to include anticollapsing otons in astrophysics on the basis of the Schwarzschild metric was made by Novikov (1964) and by Ne'eman (1965) as regions retarded in general cosmological expansion under their gravitational radius ("retarded cores").

The white hole theory has been developed in spite of certain difficulties (catastrophical accretion, quantum particle creation, the "blue layer" problem) in some measure (Narlikar and Apparao 1975, Dadhich 1977, Narlikar and Kapoor 1978, Trofimenko and Gurin 1986). That was further promoted by feasibility of immediate observability of white holes in contrast with the black ones. They can explain different explosive high-energetic cosmic phenomena : quasars, galaxy nuclei, transient X-ray sources, cosmic rays of superhigh energy (Narlikar and Apparao 1975, Narlikar *et al* 1974).

In order to develop the theory of white hole further, it has been proposed (Trofimenko 1978, 1989, Trofimenko and Gurin 1989) that the anticollapsing objects in extended STM (ESTM) should be considered.

In this alternative model of anticollapsing objects, not only the difficulties which are inherent to the "retarded cores" model is removed partially but also the sphere of possible astrophysical applications is extended (Trofimenko 1978, 1989). In the present work, the study of white and grey holes in ESTM has been continued, the dependence of blue shift for radiation from anticollapsar on event horizons is found out, correlations between event horizon and the point of maximal blue shift are obtained.

2. Anticollapsars in the Kerr-Newman extended space-time

The Kerr-Newman metric is the theoretical basis of white and grey holes-models construction. In the oblate quasispheroidal Boyer-Lindquist coordinates, it is described in the following form (here the geometrized units are used, $c=G=1$) :

$$ds^2 = -(\Delta/\rho^2)(dt-\sin^2\theta\, d\phi)^2+(\Delta/\rho^2)^{-1}\, dr^2$$
$$+\rho^{-2}\sin^2\theta\,[a dt-(r^2+a^2)d\phi]^2+\rho^2 d\theta^2, \tag{1}$$

$$\Delta = r^2-2Mr+Q^2+a^2, \tag{1a}$$

$$\rho^2 = r^2+a^2\cos^2\theta, \tag{1b}$$

where M is the total mass of oton, Q is its charge and a is its angular momentum per unit mass.

Surfaces of event horizon for the metric (eq. (1)) are defined by the expression in the usual units :

$$R_\pm = GM/c^2 \pm [(GM/c^2)^2-GQ^2/c^4-a^2/c^2]^{1/2}, \tag{2}$$

where R_+ is the external event horizon and R_- is the inner one.

Surfaces of infinite shifts are defined by the following way :

$$r_\pm = GM/c^2 \pm [(GM/c^2)^2-GQ^2/c^4-(a^2/c^2)\cos^2\theta]^{1/2}, \tag{3}$$

where r_+ is the infinite red shift surface and r_- is the infinite blue shift one. The STM region between R_+ and r_+ surfaces is named ergosphere. Structure of STM appears non-trivial due to the pseudosingular surfaces. In the case of a Kerr oton ($M\neq 0$, $a\neq 0$, $Q=0$) the picture is qualitatively is the same. When $a=0$ ($M\neq 0$, $Q\neq 0$), $r_+=R_+$ and $R_-=r_-$, i.e. event horizon surfaces coincide with corresponding shifts surfaces. Thus, the condition $a=0$ makes the STM structure even poorer, only two peculiar surfaces remain which have properties of horizons and infinite shifts surfaces. For the Kerr-Newman otons this coincidence occurs only along the symmetry axis ($\theta=0$).

Lastly, for a Schwarzschild oton the single pseudosingular surface occurs : $r_+=R_+=R_g$, R_g is the gravitational radius, which has the following value

$$2GM \tag{4}$$

The second peculiar surface ($r_-=R_-=0$) coincides with the point of true singularity. This simplest Schwarschild metric, which has poorest STM structure (Figure 1), was used for construction of the first white hole model as "retarded cores". It is not surprising therefore, that this model abstracting from such universal properties of cosmic object as rotation encounters on a number of theoretical difficulties.

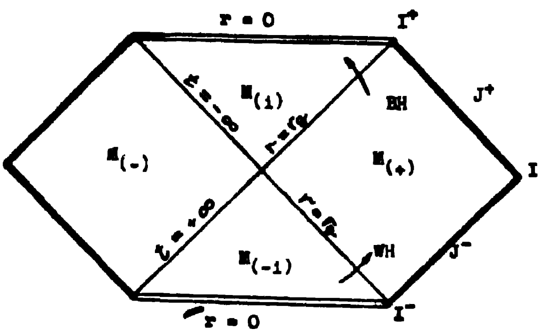

Figure 1. The Penrose diagram for the extended Schwarzschild STM. Double lines ($r=0$) mean the world lines of singularities. Thin lines ($r=r_g$) are event horizons. Points and bold lines, which are right and left boundaries of the diagram, are connected with the space-time infinites : I^+, I^-, I^0, J^+, J^- (see Misner et al 1973). The four regions of STM : I(+), II(i), III(−), IV(−i) are separated from the originate reference frame by different number of horizons (0, 1, 2, 3). Lines with arrows indicate possible geodesics (time-like) which correspond to black hole, *BH*, and white hole, *WH*.

Let us consider the Penrose diagram (Hawking and Ellis 1973) for the Kerr-Newman STM extended along the symmetry axis (Figure 2). The qualitative distinction of $M(\rho)$ on some parameter $P(M)$, is defined by the number (N) of peculiar surfaces (event horizons, light barriers separating $M(\rho)$ under consideration from the originate $M(o)$ (Trofimenko 1988) by the following way :

$$P(M)=i^N \tag{5}$$

Every region of the complete Kerr-Newman manifold can be denoted now by the general symbol $M^k_{(P)}$.

Since k is infinite, the number of regions like $M(+)$ can be infinitely large. Each region can represent itself as an independent world similar to our Metagalaxy, and bursts of white and grey holes ought to be observe evidence of multiplicity of otonic worlds. The finiteness of the fundamental invariant speed in relativity

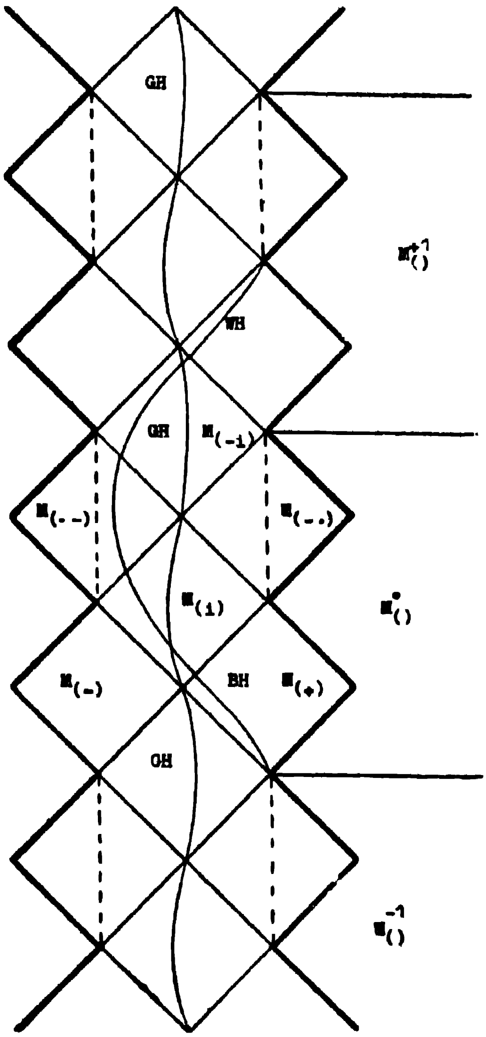

Figure 2. The Penrose diagram for the extended along the symmetry axis Kerr STM. The broken line means the ring singularity. The stencil picture $M^k(\ )$ including regions $(M(+), M(-), M(i), M(\cdot\ ), M(-\cdot)$, and $M(-i)$ is repeated unlimitedly to the both sides. When $R \to \infty$ we obtain the complete Kerr-Newman STM Curves show possible geodesics (time-like) which correspond to black hole, *BH*, white hole, *WH* and grey hole, *GH*.

($V = c =$ const. $< \infty$) is the fundamental base of appearance in *GR* of ESTM with unusual structure (Figure 2) as well as of the idea of variety of otonic worlds themselves. From the finiteness fundamental speed, the possibility of causally non-connected space-time regions follows, multiplicity of which is just interpreted as variety of otonic worlds (Metagalaxies). Transition to non-relativistic limit $c \to \infty$ is the single general foundation of reduction of such STM structures. Let us show the same using the example of Kerr-Newman metric (eq. (1)).

At $c \to \infty$ it is easily seen that both event horizons (eq. (2)) and infinite shift surfaces (eq. (3)) disappear, i.e. all the unusual structure of the Kerr-Newman space-time vanishes. Conditions $Q=0$ and $a=0$ make poorer STM structure, but they do not reduce it to the Minkowski space-time.

Existence of causally non-interconnected STM regions is equivalent to existence of additional independent spatial and temporal coordinates which means increase of the total number of coordinates as well as of the global space-time dimensionality overlaping these worlds (Trofimenko and Gurin 1986, 1987, 1988). Thus, white holes are indirect evidence of many-dimensional space-time structure of the universe as "windows from higher-dimensionalities".

Anticollapsing objects in similar ESTM are formed as the result of relativistic collapse-anticollapse process from black hole matter, which "flows" (Figure 1) through wormholes from one (M^0_+, M^0_i) ESTM region (otonic world) to another (M^1_{-i}, M^1_+). The cause of transformation of collapse to anticollapse consists in rotation for the Kerr oton. Rotation leads it to expansion at the certain step of contraction, namely, in the region (M^0_{-}) at $R = R_0 = a^2/c^2R_b$. Therefore, otonic model of white holes does not require a special introduction of negative C-field in order to base collapse-anticollapse process: rotation plays ($a = L/M$) its role. Thus in the concept of white hole, we should go from the Schwarzschild STM to the Kerr ESTM, which explains the nature of expansion quite naturally and leads to the notion on non-trivial ESTM structure, and on worlds variety.

Grey hole (oton) is a theoretical construction like cosmic oscillator (Narlikar 1974, Apparao and Narlikar 1982). So, black hole (regions M^0_+, M^0_i) is associated with the passive stage in life of the cosmic oscillator (oton) when the accumulation of energy and matter dispersed in original space occurs ; white hole corresponds to the active (regions M^1_{-i}, M^1_+) stage when otonic matter saturated by energy is expanded impetuously (fragmentized).

The otonic white holes-model allows to remove some difficulties connected with the idea of "retarded cores" (Trofimenko and Gurin 1989). Otonic white hole does not have a retardation time ; it anticollapses just from the moment of its appearance in a concrete otonic world ; i.e. at the beginning in M^1_+ there is no a stationary gravitational field in which effects of the "blue layer" and the creation of quantum particle leading to the self-closing are possible.

The frequency shift is inherent for both the given model and for all other ones. It leads to spectral features of anticollapsing objects.

3. Spectral features of radiation from white and grey holes

Since white holes must be rather bright objects, radiation from their surface can have infinite blue-shift, which however, occurs for a very short time. The following spectral features of anticollapsar in three cases of the Kerr-Newman white hole are considered in the work by Dadhich (1977) : (1) Reissner-Nordström anticollapsar, (2) Kerr-Newman anticollapsar in the equatorial plane, (3) Kerr-Newman anticollapsar along the symmetry axis ($\theta=0$).

These three cases can be represented in the general form

$$\frac{\nu}{\nu_0}=(f_b)^{1/2}\pm(f_b-f)^{1/2} \tag{6}$$

where, $f=f(r)=g_{00}$ is the metrical coefficient of the time coordinate in corresponding metric ; $f_b=g_{00}$ $(r=R_b)$; ν_0 is the frequency of signal emitted radially from the white hole surface ; ν is the frequency of light signal received by a remote observer.

The expressions for ν (Dadhich 1977) do not show explicitly the role of horizons in white hole radiation, though their importance is quite known for black holes ; at their horizon the radiation has the infinite red-shift and under horizon it is unobservable. In order to show the role of horizons in anticollapsar's radiation we represent eq. (6) as follows :

$$\frac{\nu}{\nu_0}=\left[\frac{(R_b-R_+)(R_b-R_-)}{\rho_b^2}\right]^{1/2}\pm\left[\frac{(R_b-R_+)(R_b-R_-)}{\rho_b^2}-\frac{(r-R_+)(r-R_-)}{\rho^2}\right]^{1/2} \tag{7}$$

Depending on the value of R_b with respect to R_+ one can pick out the four types of anticollapsing otons with their features of radiation.

(1) Note that the ideal (canonical) white hole must be parabolic ($R_0=\infty$), since in opposite case we have the oscillating collapse ($R_0 < R_b < \infty$), i.e, one kind of grey holes. For the parabolic white hole we have from eqs. (6) and (7)

$$\frac{\nu}{\nu_0}=1\pm(1-f)^{1/2}=1\pm\left[1+\frac{(r-R_+)(r+R_-)}{\rho^2}\right]^{1/2}, \tag{8}$$

$$f=1 \text{ when } R_0=\frac{R_+R_- -(a/c)^2}{R_0}$$

(R_0 is the point from which expansion begins) ν is real for all $r\geqslant\frac{R_+R_- -a^2/c^2}{R_0}$.

(2) The ideal (canonical) grey hole corresponds to the condition $R_b = R_+$. For the canonical grey hole we have from eqs. (6) and (7)

$$\frac{\nu}{\nu_0} = \pm(-f)^{1/2} = \left[-\frac{(r-R_+)(r-R_-)}{\rho^2}\right]^{1/2} \tag{9}$$

is real during all the anticollapse $R_0 = R_- \leqslant r \leqslant R_+ = R_b$. The peculiar character of canonical grey holes lies in the stability of oscillations. The light-grey hole ($R_+ < R_b < \infty$) must be fragmented at local inhomogeneities. As for the Kerr oton, oscillations $R_b > R_+$ are more difficult, since the gravitational field itself becomes the factor which destroys initial structure. The dark-grey holes are localized in the region $M^k_{(-i)}$ and can accumulate energy up to the level of the canonical grey hole.

(3) The condition $R_+ < R_b < \infty$ corresponds to the light-grey hole to $0 < f_b < 1$ at $f_b > f$, ν is real, $R_0 < R_- < R_+ < R_b$.

(4) The condition $R_b < R_+$ corresponds to the dark-grey hole. There is the constant imaginary component for ν, since $f_b < 0$. The real component of $(f_b + f)^{1/2}$ for $f_b > f$ can be infinite when $Q \to 0$ or $a \to 0$ and the mainfestation time is infinitely small, $R_- < R_0 < R_b < R_+$. The dark-grey hole in contrast with the light-grey one, which intersects horizons and passes different ESTM regions, $M^1_{(+)}$, $M^1_{(i)}$, $M^1_{(-)}$, $M^2_{(-i)}$, $M^2_{(+)}$ ..., and oscillates in the region $M^k_{(-i)}$ until it become the canonical grey hole when it attains the event horizon.

From the above and the expression (7) it is clear that the peculiar character of horizons reveals for anticollapsars not for the point from which radiation emerges but for R_b, i.e. the parameter of maximal extension.

Let us find out the connection of the horizons with other parameters of otons.

We represent eq. (1a) in another form by introducing of a quantity like the classical radius $R_e = Q^2/Mc^2$ and, using eq. (4) and $a = 0$, as follows :

$$f = g_{00} = \frac{\Delta}{r^2} = 1 - \frac{R_g}{r} + \frac{R_g R_e}{2r^2}. \tag{10}$$

The condition $\Delta = 0$ determines horizons, for which we obtain from eq. (10)

$$R_\pm = \frac{R_g}{2} \pm \left[\frac{R_g}{2}\left(\frac{R_g}{2} - R_e\right)\right]^{1/2} \tag{11}$$

and from eq. (11)

$$R_+ + R_- = R_g \; ; \; R_+ R_- = \frac{R_g R_e}{2}. \tag{12}$$

the expressions (12) leads to

$$R_e = 2R_+R_-(R_+ + R_-)^{-1}, \tag{13}$$

i.e. R_a is the harmonic mean of R_+ and R_-. Moreover, from the Dadhich's (1977) expression for the classical radius we have, $R_o = 2R_bR_a(R_b+R_a)^{-1}$, i.e. where R_o is the harmonic mean of R_b and R_0.

Introducing the expression $(R_gR_a/2) = a^2/c^2$ for the Kerr oton ($\theta = 0$), one can obtain formulae analogous to eqs. (10)-(13). Taking into account the correlation $R_m = \pm a/c$ and $R_0 = a^2/c^2R_b$ we can write the following

$$(a^2/c^2) = R_m^2 = R_0R_b = (R_gR_a/2) = R_+R_-. \tag{14}$$

The points R_m (the maximal blue shift point) as seen from eq. (14) should be the geometrical mean of R_+ and R_- and also of R_b and R_0. From eqs. (11)-(14) it follows that $R_0 < R_+ < R_g$, i.e. initial sizes of anticollapsar are smaller than R_g when $R_m \leqslant R_g$ and $R_0 \leqslant R_g$.

For the charged and rotating (we consider here the case of $\theta = 0$ only) canonical white hole ($R_b = \infty$) we have the same values in the corresponding points : $\nu = 2\nu_0(R_-, R_+)$, $\nu = \nu_0(R_0, R_b)$. The expression for ν_{max} has the following form

$$\nu_{max} = \nu_0[1 - (R_g/R_m)^{1/2}], \tag{15}$$

both for charged and rotating canonical grey hole one follows $r = R_0 = R_-$ from the condition $r = R_b = R_+$. At these points we have the infinite red shift $\nu = 0$. The expression for ν_{max} has the following form

$$\nu_{max} = \nu_0\left(\frac{R_g}{2R_m} - 1\right)^{1/2}. \tag{16}$$

The value R_m for the charged grey (white) hole is determined by eq. (13) and for the rotating one by eq. (14).

It may be noted that though the rotation and charge weaken the blue-shift, it can also be large enough for these cases. Let us give the explicit expressions for ν_{max} via the global parameters.

White hole :

(1) $Q \neq 0$, $a = 0$. (2) $Q = 0$, $a \neq 0$, $\theta = 0$.

$$\nu_{max} = \nu_0\left[1 + \frac{M(G)^{1/2}}{Q}\right], \quad \nu_{max} = \nu_0\left[1 + \left(\frac{GM}{ca}\right)\right]^{1/2}. \tag{17}$$

Grey hole :

(1) $Q \neq 0$, $a = 0$. (2) $Q = 0$, $a \neq 0$, $\theta = 0$

$$\nu_{max} = \nu_0[(M^2G/Q^2) - 1]^{1/2}, \quad \nu_{max} = \nu_0[(GM/ca) - 1]^{1/2}. \tag{18}$$

From the expressions (15)–(18) we observe that the bigger is the mass and the smaller is charge (rotation), the more the maximal blue-shift of radiation from anticollapsar. Thus the stronger is the action of gravitation, the more is the blue-shift, i.e. gravitational interaction has a character of antigravity. Thus, although charge and rotation make the blue-shift weaker, for $M(G)^{1/2} \gg Q$ and $GM \gg ca$, $\nu_{max} \gg \nu_0$, i.e. it can be considerably large. In the Schwarzschild case ($Q=0$, $a=0$), we obtain $\nu_{max}=\infty$, that coincides with the Kerr anticollapsar in the equatorial plane. Therefore, anticollapsars give broad spectrum of possibilities for model construction of phenomena of high-energy astrophysics.

For light (dark)-grey hole at the points R_+, R_-, R_b, R_m we have,

(1) $r=R_+$, $r=R_-$

$$\nu_\pm = 2\nu_0[(R_b-R_+)(R_b-R_-)(R_b^2+R_m^2)^{-1}]^{1/2}, \tag{19}$$

when

$Q=0$, $a\neq 0$

$$\nu_\pm = 2\nu_0[1-R_g(R_b+R_0)^{-1}]^{1/2}. \tag{20}$$

For light-grey hole $\nu_\pm$ is real, for dark-grey it is imaginary

(2) $r=R_b$

$$\nu_b = \nu_0[(R_b-R_+)(R_b-R_-)(R_b^2+R_m^2)^{-1}]^{1/2}, \tag{21}$$

when

$Q=0$, $a\neq 0$

$$\nu_b = \nu_0[1-R_g(R_b+R_0)^{-1}]^{1/2}. \tag{22}$$

For light-grey hole ν_b is real, for dark-grey hole it is imaginary

(3) $r=R_m$

$$\nu_m = \nu_0[(f_b)^{1/2}+(f_b-f_m)^{1/2}], \tag{23}$$

when

$Q=0$, $a\neq 0$

$$\frac{\nu_m}{\nu_0} = \left(1-\frac{R_g}{R_b+R_0}\right)^{1/2} + \left(\frac{R_g}{2R_m}-\frac{R_g}{R_b+R_0}\right)^{1/2}. \tag{24}$$

For light-grey hole ν_m is real, for dark-grey Kerr hole due to the inequality $2R_m \leqslant R_b+R_0$, ν_m has an imaginary component.

4. Discussion

Within the framework of the otonic worlds idea (Trofimenko 1978), the following cosmic phenomena are associated with white and grey holes (Trofimenko 1989) : "voids" in the Universe as white hole relics ; gamma-bursts considered as grey hole manifestation ; bursts of neutrinos, gravitational waves, cosmic rays ; chemical elements, "hidden mass" of the Universe ; initial inhomogeneities.

The discovery of a white hole or grey hole explosion is essentially significant, which is however a rather complicated experimental problem due to the transient character of the blast non-recurrences and its manifestation in the region of super-high energies. If gamma-bursts are connected with grey hole explosions, one can predict the occurrence of synchronised bursts of gravitational radiation. Bursts of gravitational radiation have been registered by Weber (1969), but from the energy-considerations these results do not justify the discovery of gravitational waves. The occurrence of the gravitational waves bursts is the principal test which makes distinction between white and grey holes from another otonic worlds and "retarded cores" explosions. "Retarded cores" have the static gravitational field in the external STM and the core expansion (even in the case of axial symmetry) has not led to the generation of gravitational waves. Perhaps white and grey holes from another otonic worlds causing extreme perturbations of STM and gravitational field may lead to bursts of gravitational radiation.

Acknowledgments

In conclusion, the author expresses gratitude to Profs Narlikar J V and Penrose R for remarks on the question of white holes, Profs Weber J and Kapoor R C for sending the reprints ; Profs Idlis G M and Efremov J N for countenance of this work ; Gurin V S and other participants of the seminar on relativistic astrophysics for constructive discussion on the present work.

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